Using *Sun Moon Planets* - Orbital Elements

For any object orbiting around the Sun, its real orbit changes over time due to gravitational perturbations and the effects of relativity. By ignoring these physical effects and treating the orbit in a strictly mathematical sense we can specify a set of orbital elements used to describe the position of an object such as a planet, asteroid, or comet as it orbits around the Sun. In this arrangement there are only the two bodies, the object and the Sun; there are no other external physical influences on the motion of the object.

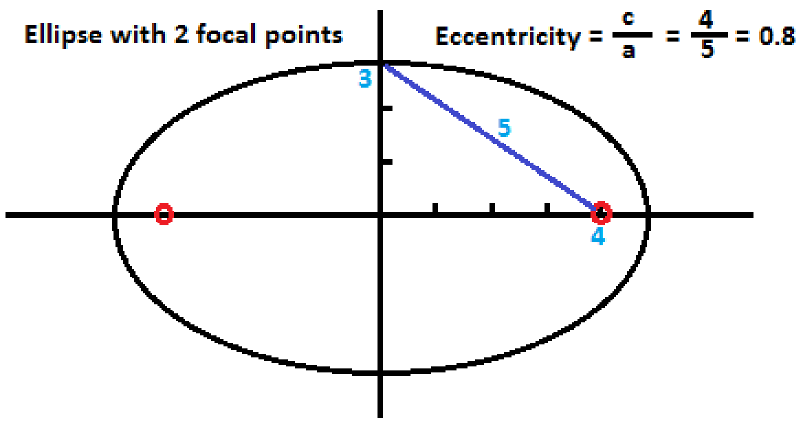
These types of orbits considered in this program are described as classical two-body systems and are based on the six Keplerian elements:

- **Eccentricity (e)** - the shape of an ellipse as compared to a circle. For a circle, $e = 0$; e increases as the circle becomes more elliptical. If $e = 1$ the orbital path is parabolic and if $e > 1$ it is a hyperbola (these geometric shapes are not considered in this program).
- **Semi-major axis (a)** - add the perihelion and aphelion distances and divide by 2. This is the average distance between the Sun and the object.
- **Inclination (i)** - this is the tilt of the ellipse with respect to the ecliptic. It is measured at the ascending node.
- **Longitude of ascending node (Ω)** - this is the point where the orbiting object passes upward through the ecliptic.
- **Argument of perihelion (ω)** - the perihelion is the closest point that the object comes to the Sun. The argument of perihelion is the angle from the ascending node to the point of perihelion.
- **Mean anomaly (M)** - this is the angle from the perihelion to a point along the path of the orbit. The angle is based on the mean average speed of the object and the amount of time it has been travelling on that orbital path since it left the point of perihelion. The mean anomaly is only a guess at where the object actually is because the speed of the object is not constant - it varies as the object orbits around the Sun.

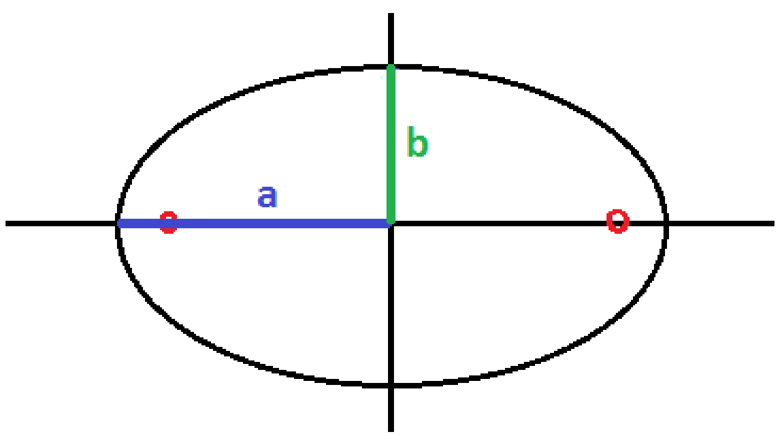
The mean anomaly is often specified at a particular time and date (epoch). The mean anomaly at epoch states the position of the orbiting object along the ellipse at a specific time.

- **True anomaly (v)** is not usually specified as one of the six orbital elements. The true anomaly is the actual position of the object along the orbital path. The true anomaly is solved by using **Kepler's Equation (M = E - e sin E)** where **E** is called the **Eccentric anomaly**.

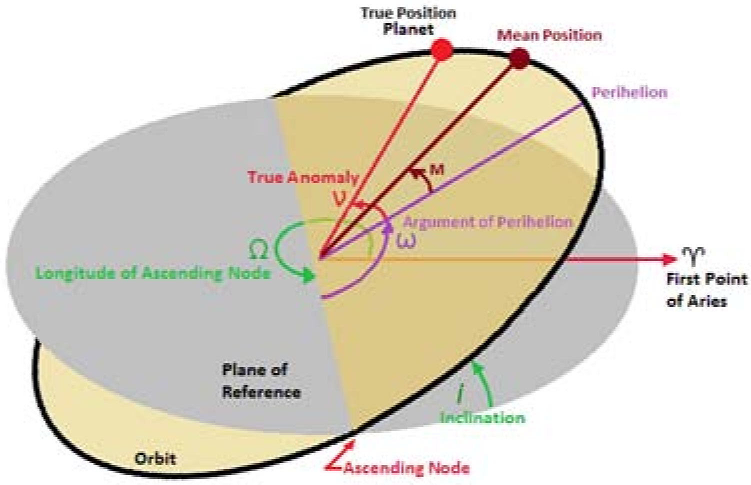
Eccentricity (e) of an Ellipse



Semi-Major Axis (a) and Semi-Minor Axis (b)

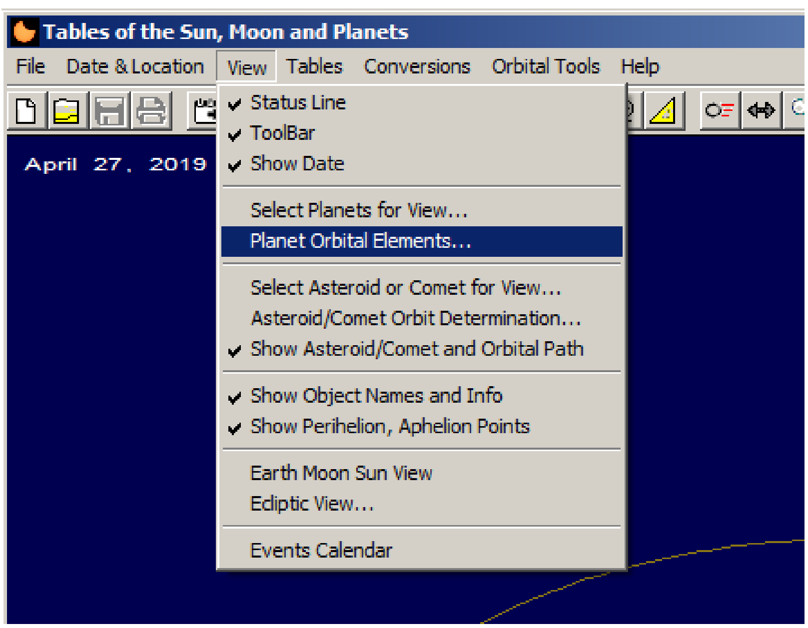


Inclination (i), Longitude of Ascending Node (Ω), Argument of Perihelion (ω), Mean Anomaly (M), and True Anomaly (ν)

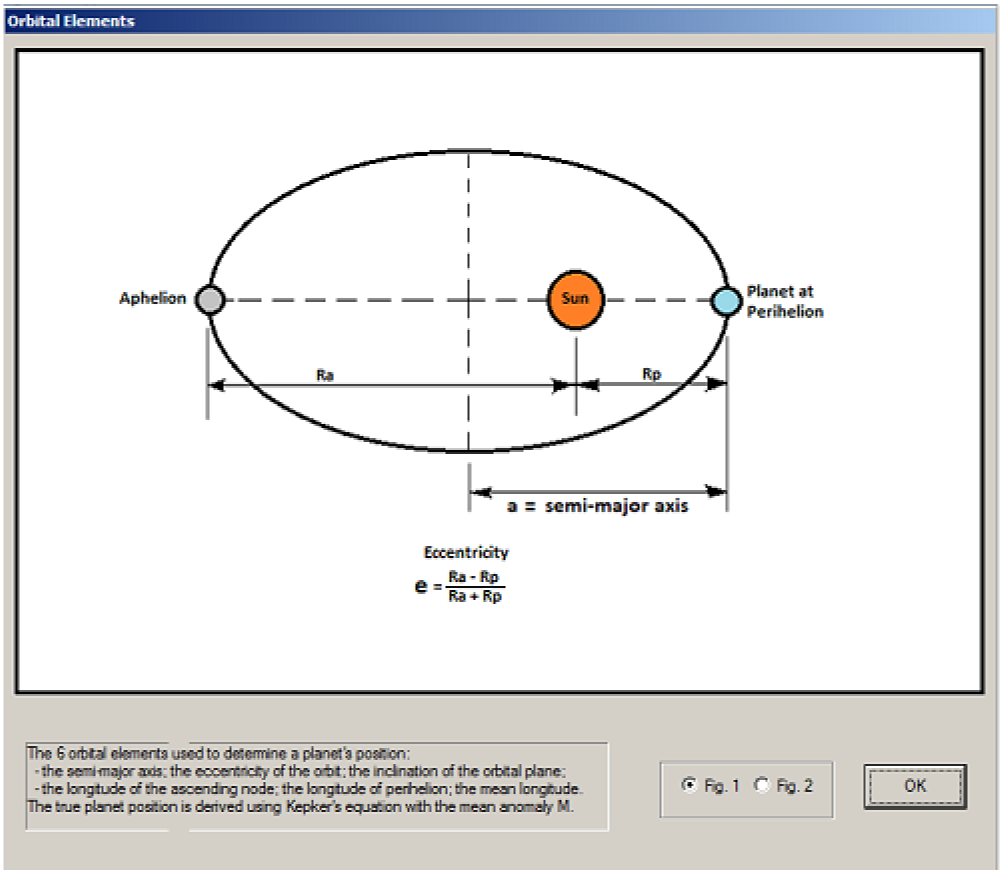


Now let's try to see these orbital elements as they are used in ***Sun Moon Planets***

Go to the **View** menu and select **Planet Orbital Elements**.

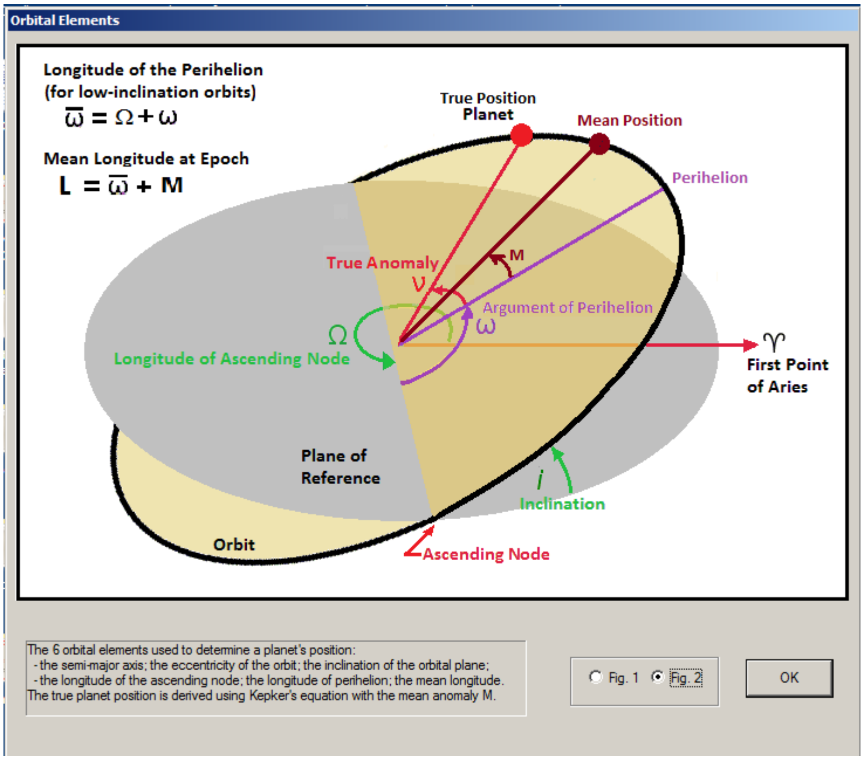


From that dialog box click the **button '?'** at bottom right, and then select the radio button for **Fig. 1** image (see below). The image shows two orbital elements, the **Semi-Major Axis (a)** and the **Eccentricity (e)**.



Note the different way of calculating eccentricity than the method shown above.

Next, from the dialog box select the radio button for **Fig. 2** image (see below). The image shows several orbital elements, which we will now look at in detail.



Notice the red line starting at the center and pointing to the right towards the **First point of Aries** (**Vernal Equinox).** This is the point at which the Sun crosses the Celestial Equator as it moves from the south to the north along the ecliptic.

Starting at the **First Point of Aries** observe the green arc going around to the **Ascending Node**. This is **O**mega (**Ω**), the **Longitude of the Ascending Node.** Sometimes, instead of **O**mega or **Ω** the letter **N** is used. Think **N** for **Node**.

Next, observe the arc from the **Ascending Node** to the **Perihelion**. This is **o**mega (**ω**). It is called the **Argument of Perihelion**. If you add those two together (**Ω** + **ω**) you get the **Longitude of the Perihelion** (**ϖ**).

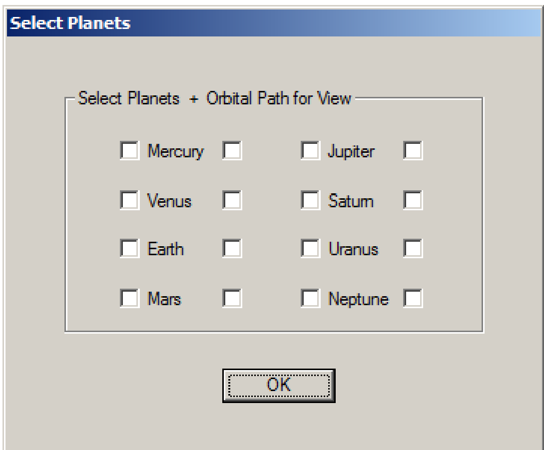
Next, observe the arc from the **Perihelion** to the **Mean Position**.
This angle **M** is called the **Mean Anomaly**. Now if you add all three together you get the **Mean Longitude** (**L**) at the **Date of Elements**. The **Date of Elements** is often called the **Epoch**.

Note that the **Mean Position** is fairly close to the **True Planet Position**.

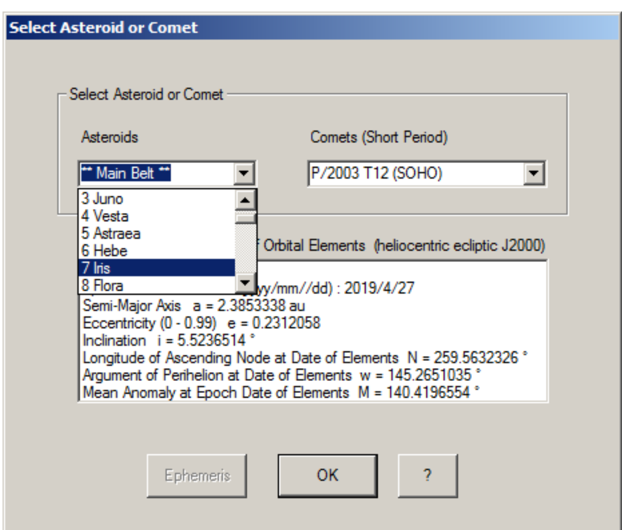
This angle is called the **True Anomaly** and is solved by using **M** with **Kepler's Equation.**

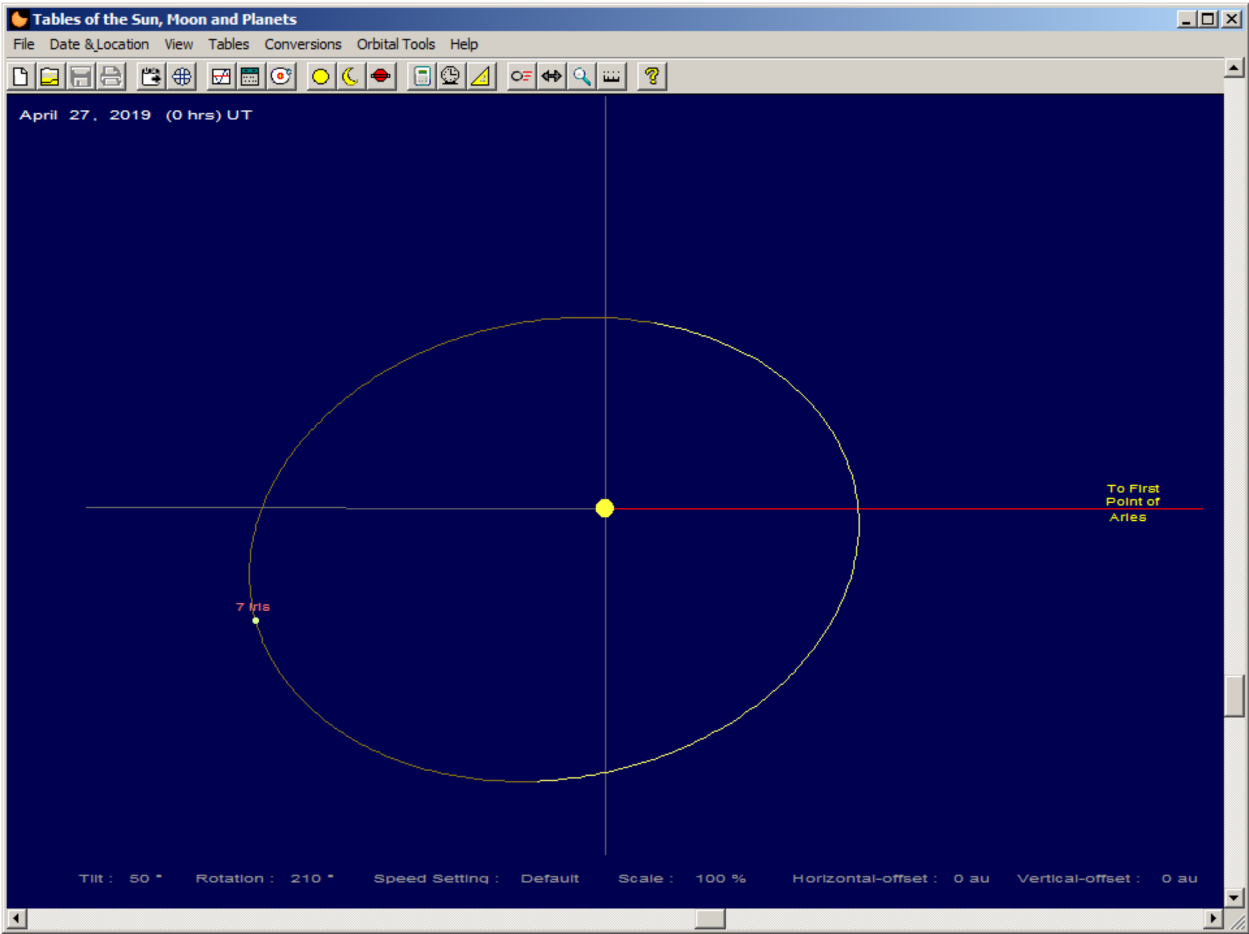
Now lets apply these ideas to an asteroid or comet

First, use the **View** menu and click on **Select Planets for View**. In that dialog box uncheck all the planets and their orbital paths.



Next, use the **View** menu item and click item **Select Asteroid or Comet for View**. In the dialog box select the 7th asteroid, **Iris**. You should now see only the orbit of Iris on the screen. Without the planets, it is easier to study the orbit.



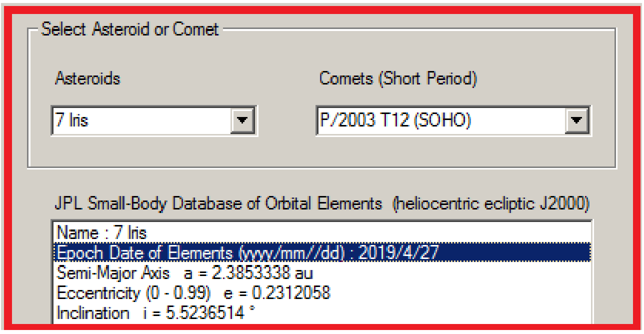


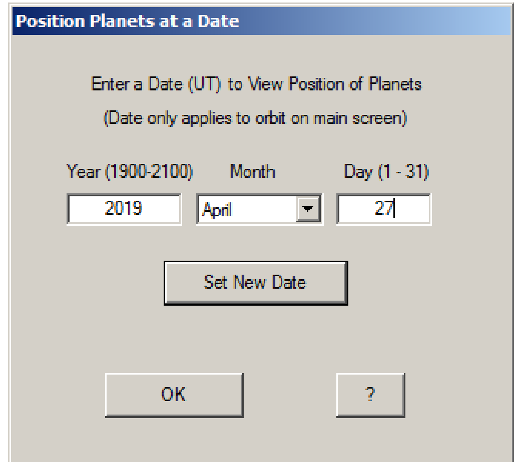
Note how it is somewhat similar to the orbital image previously in **Fig 2**.

In the dialog box for asteroid **Iris**, note the **Semi-Major Axis (a)** is 2.3853338 au. This means that on average the asteroid is more than twice as far from the Sun as Earth (Earth is 1 au).

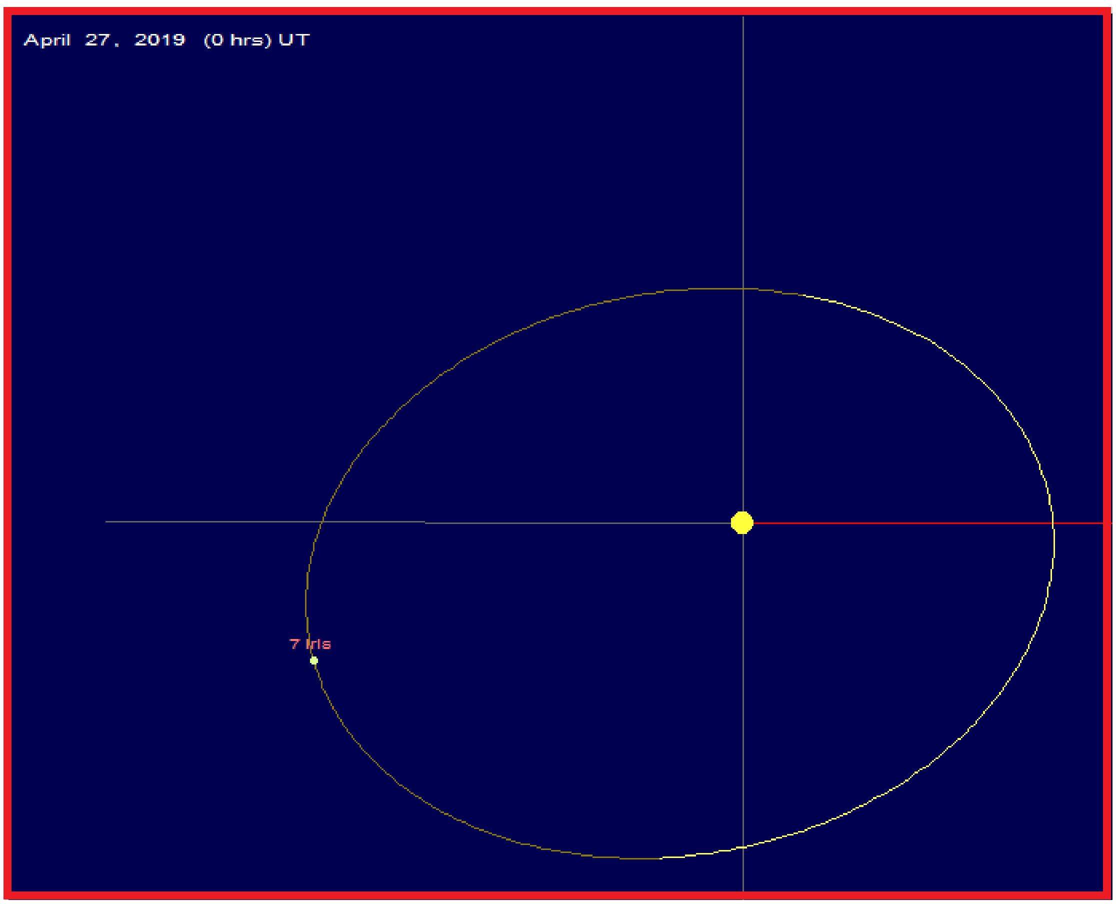
Also note in the dialog box that the **Eccentricity (e)** of **Iris** is only 0.2312058. It's definitely not a circle, but neither is it very elongated as many comets are.

Now use the **Date & Location** menu item to set the screen date to the **Epoch Date of Elements** in the dialog box. As you can see this is **2019/4/27**.





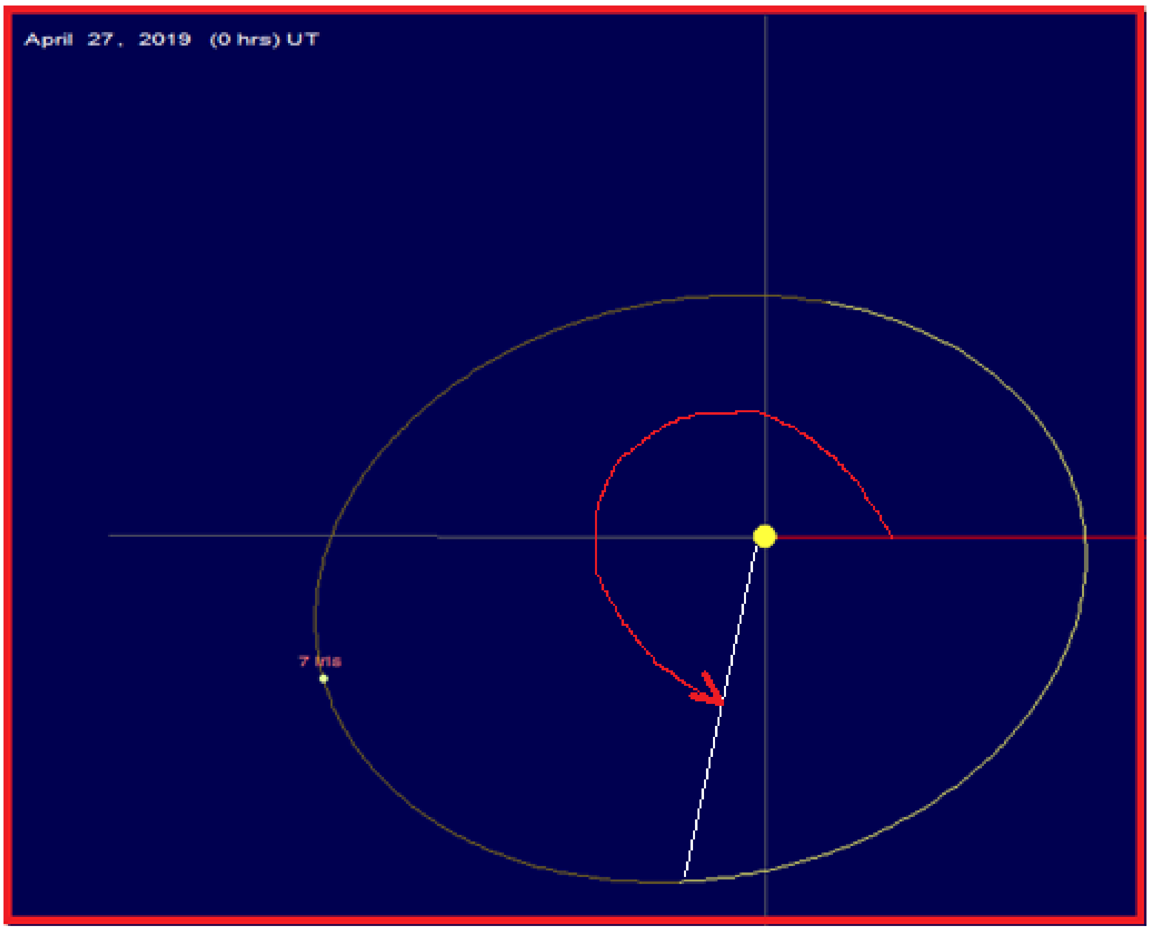
Now you should see the orbit of **Iris** and its position as of April 27, 2019.



The bright part of the orbital path as shown on the screen indicates where the orbit is above the ecliptic plane. The darker part of the orbital path indicates where the orbital path is below the ecliptic plane.

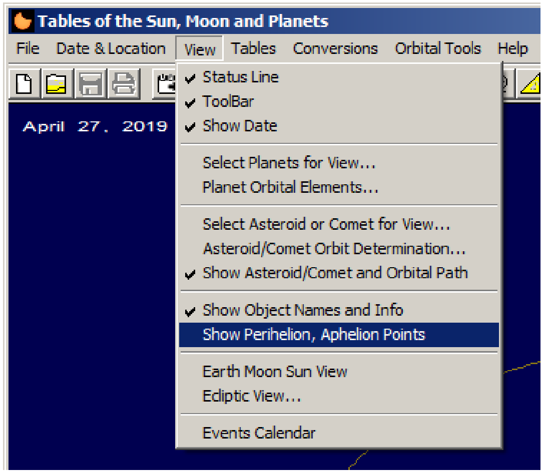
Now, imagine drawing a line from the Sun to the **Ascending Node** (near the bottom of the image where the darker orbital path changes to the brighter orbital path (the opposite node is descending since objects orbit around the Sun counter-clockwise from the top view).

Next imagine drawing an arc starting at the red line (**First Point of Aries**) around to the line from the Sun, as shown below.

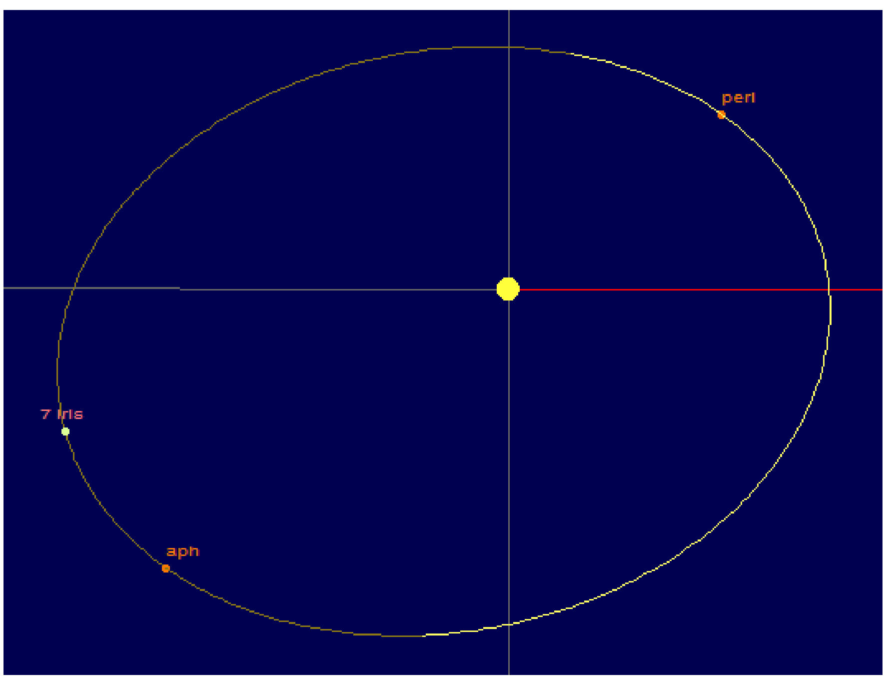


The angle of that arc is called the **Longitude of Ascending Node** and in this case for asteroid **Iris** you can see in the table of the asteroid dialog that the **Ascending Node** (**Ω** or **N**) is 259.5632326°. This makes sense since a full circle would be 360°.

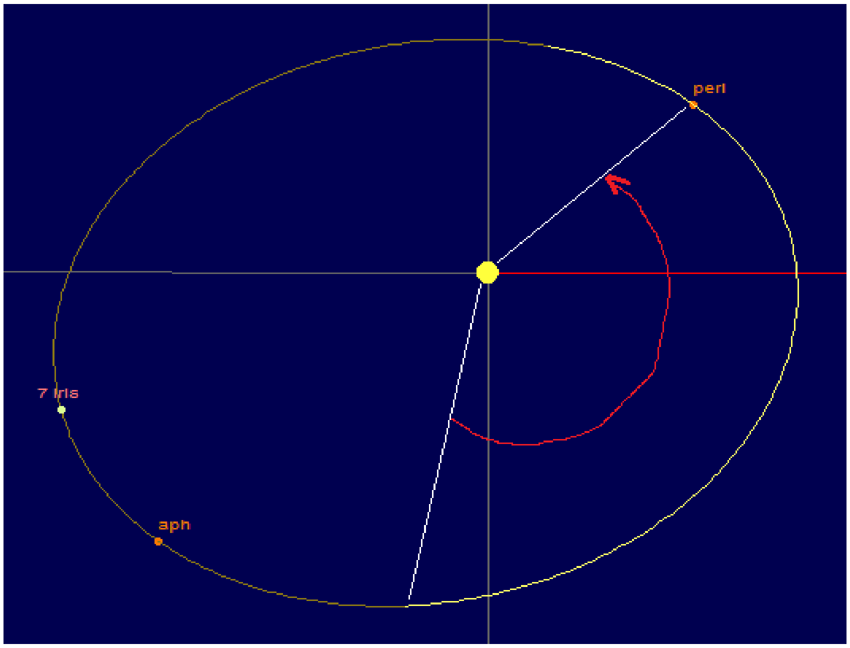
Now imagine drawing another arc from the **Ascending Node** to the **Perihelion**. The **Perihelion** of the orbit can be found by going back to the **View** menu and clicking **Show Perihelion, Aphelion Points**.



The screen will then show the following.

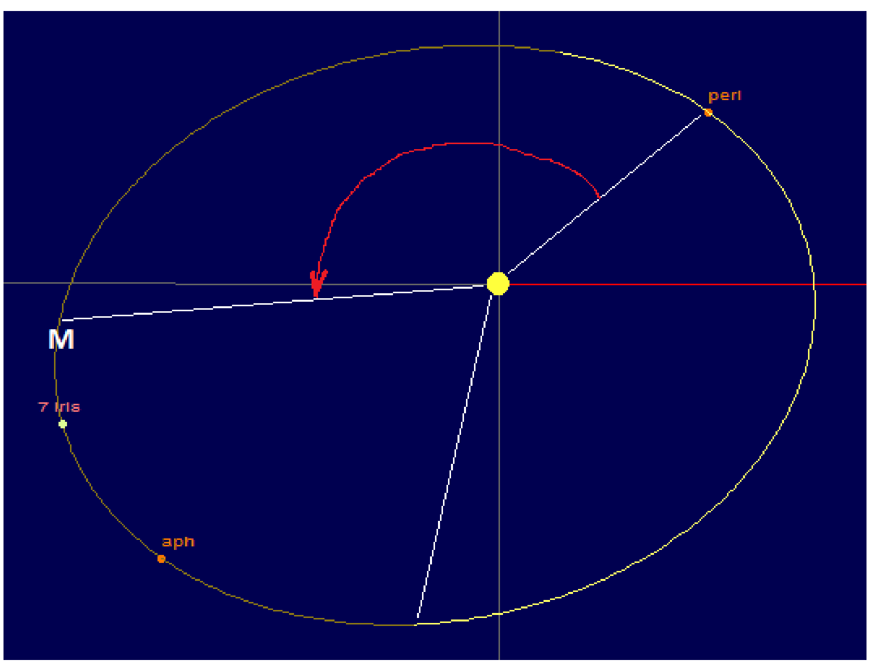


Note the two points on the orbital path marked **'peri'** and **'aph'**.
Now, imagine drawing a line from the Sun to the point of **Perihelion** and then drawing an arc from the **Ascending Node** to that line. That arc or angle is called the **Argument of Perihelion** (**o**mega or **ω** or sometimes just **w**).



If you look in the table on the asteroid dialog box you will see that the **Argument of Perihelion** is (**o**mega or **ω** or **w**) is 145.2651035°. This appears to be correct (the arc covers approx 45° + 90° + approx 10° = 145°).

The next step is to draw a line from the Sun to an imaginary point on the orbital path called the **Mean Anomaly** (**M**). Then draw an arc from the **Perihelion** to that line. That arc or angle is called the **Mean Anomaly** (**M**).



By checking the table in the dialog box we see that the **Mean Anomaly** (**M**) for **Iris** is 140.4196554° (at the Epoch date of 2019/4/27). This appears to be correct (the arc covers approx 45° + 90° + approx 5° = 140°).

From **Fig. 2** we can see that the **Mean Anomaly** is close to the **True Anomaly** (**True Planet Position**) but not exactly. The **Mean Anomaly** is based on an *average motion per day* times the number of days since the asteroid passed the **Perihelion**. The problem is that according to **Kepler's Laws**, an orbiting object does not have a constant speed - it goes fast and slow depending on where it is. Therefore we just use an average speed - and that gives us the **Mean Anomaly**. The **True Anomaly** is derived by solving **Kepler's Equation**.

Now you can see how the three angles **Ω** + **ω** + **M** give us the **Mean Longitude** at a particular date (**Epoch**). It all started from the **First Point of Aries** (**Vernal Equinox**).

Throughout all of this you need to keep in mind that the **Mean Anomaly** above was calculated at the **Epoch** of **2019/4/27 at 0 hours UT**. As the days advance the asteroid continues to move along the orbital path and so these additional days must be included in the calculation of the **Mean Anomaly**.

When you combine these three orbital elements (**Ω , ω , M**) with the other three orbital elements (**Semi-Major Axis a, Eccentricity e, Inclination i**) you have a full set of coordinates to describe the heliocentric position of any orbiting object around the Sun.

<u>A Bit of Math Regarding the Mean Anomaly</u>

Let **te** represent the **Epoch Date** and **tp** represent the time and date at which the asteroid (or comet) is passing the point of **Perihelion** (the point closest to the Sun). From these two dates we calculate the number of days from **Perihelion** to the **Epoch** ie., number of days = **te - tp** .

Also, let **n** represent the *average speed* that the object is moving (in terms of **degrees per day**). It is also called **Mean Daily Motion**.

Let **Mo** represent the **Mean Anomaly**. This is the angle (in degrees) that the object has traveled from the time of the **Perihelion** (**tp**) to the time of the **Epoch** (**te**). Use the equation **Mo = n * (te - tp)** to calculate the **Mean Anomaly**.

Mo is the value of **M** at **Epoch**.
tp is the time of **Perihelion** passage (in Julian Day Number).
te is the number of days from **tp** to the **Epoch Date.**
n is the average speed (degrees per day) or **Mean Daily Motion**.

The **Period** (**P**) of the orbit is the number of days it takes for the object to make a complete orbit around the Sun. The Period can be calculated using one of Kepler's Laws: The square of the period **P** of any planet is proportional to the cube of the semi-major axis **a** of its orbit. Therefore solve for **P** given $\mathbf{P^2 = a^3}$
When we calculate **P** in terms of Earth's orbit : **P** = **365.256898326 *** $\mathbf{a^{3/2}}$ days.
If we know the semi-major axis **a** of an object we can solve for **P**. Or, we can just count the number of days for a complete orbit.

Now that we have **P**, we can solve for the **Mean Daily Motion** (**n**). Since an object moves through 360 degrees in one orbit then the **Mean Daily Motion** is
n = 360 / Period (degrees per day).

Therefore, **Mo = (360 / Period) * (te - tp)**

Epoch is the date at which all the orbital elements were last calculated (in Julian Day Number). **JPL** (Jet Propulsion Laboratory) sets the **Date of Epoch** at which **Mo** is calculated. In the above case, JPL set the **Epoch Date** at **2019/4/27**.

If we want to know the **Mean Anomaly** (**M** not **Mo**) at a later date (the object has moved farther along its orbit) then we use the same type of calculation.

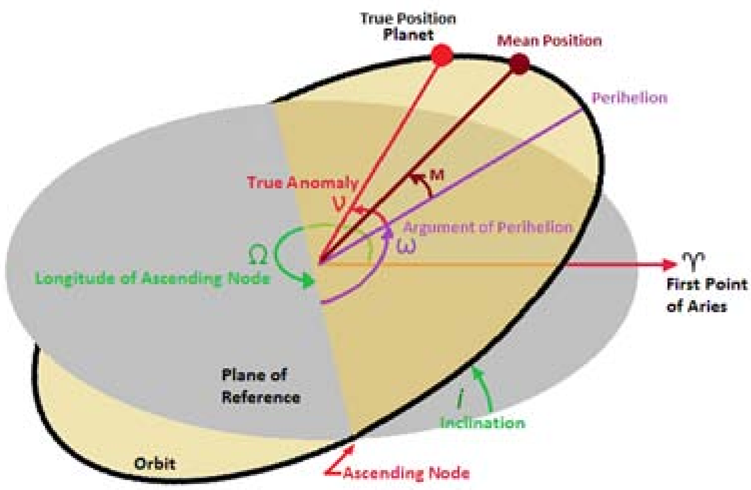
M is the **Mean Anomaly** at any arbitrary date. Instead of **(te - tp)** we use **(t - te)** where **t** is typically the present date (or any arbitrary date).

Then at any arbitrary date, **M = n * (t - te) + Mo** where again **t** is typically the present date. If we combine **(t - te)** and **(te - tp)** then we can skip **Mo** and say :

M = n * (t - tp).

One last detail; suppose we let the **Epoch Date** be the same as the **Perihelion Date** (ie. **te = tp**). This means that since **Mo = n * (te - tp)** and **te - tp = 0** then **Mo = 0**. In this case you would use **M = n * (t - tp)**.

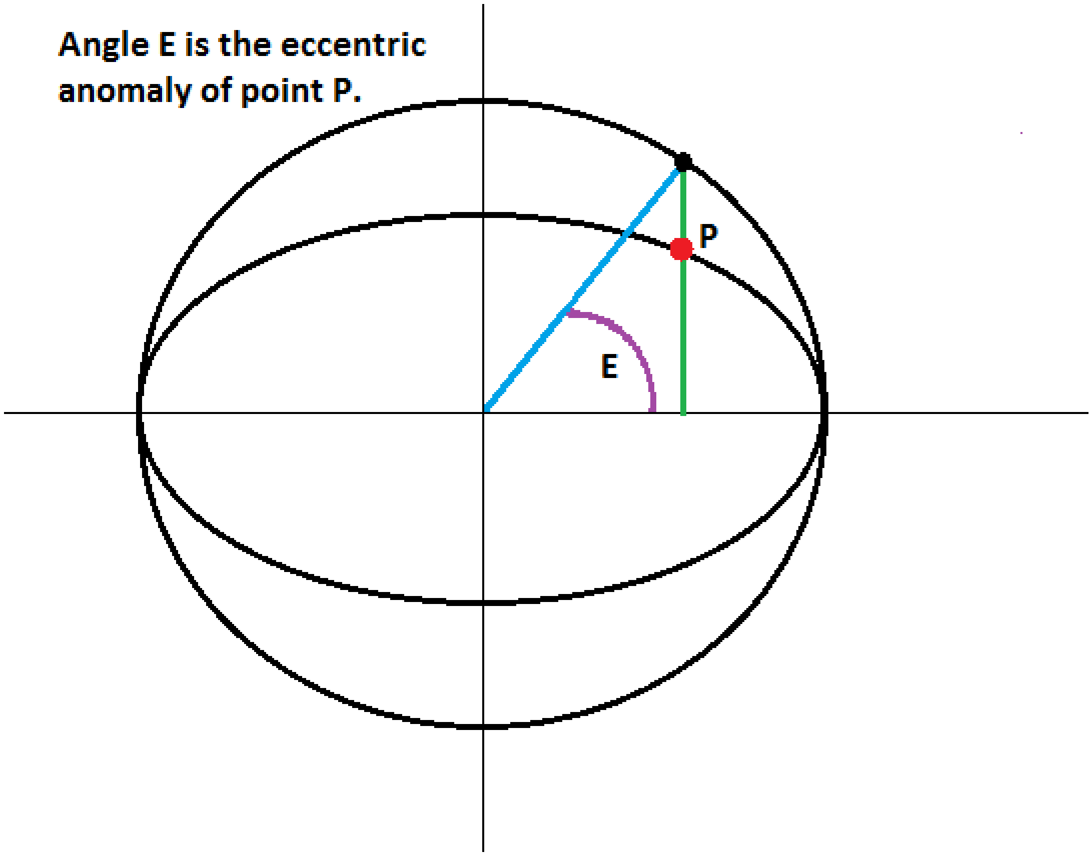
Now that we understand the **Mean Anomaly**, let's move on to the **True Anomaly**; the actual position of the object.



Before we solve for the **True Anomaly (v)** we need to understand the **Eccentric Anomaly (E)**.

In the figure below, imagine the elliptical path of an orbit with planet P, all enclosed by a circle. Draw a line from the center up to the circle such that a triangle is formed with the vertical side passing down through the planet P.

The angle **E** that is formed is then called the **Eccentric Anomaly**.

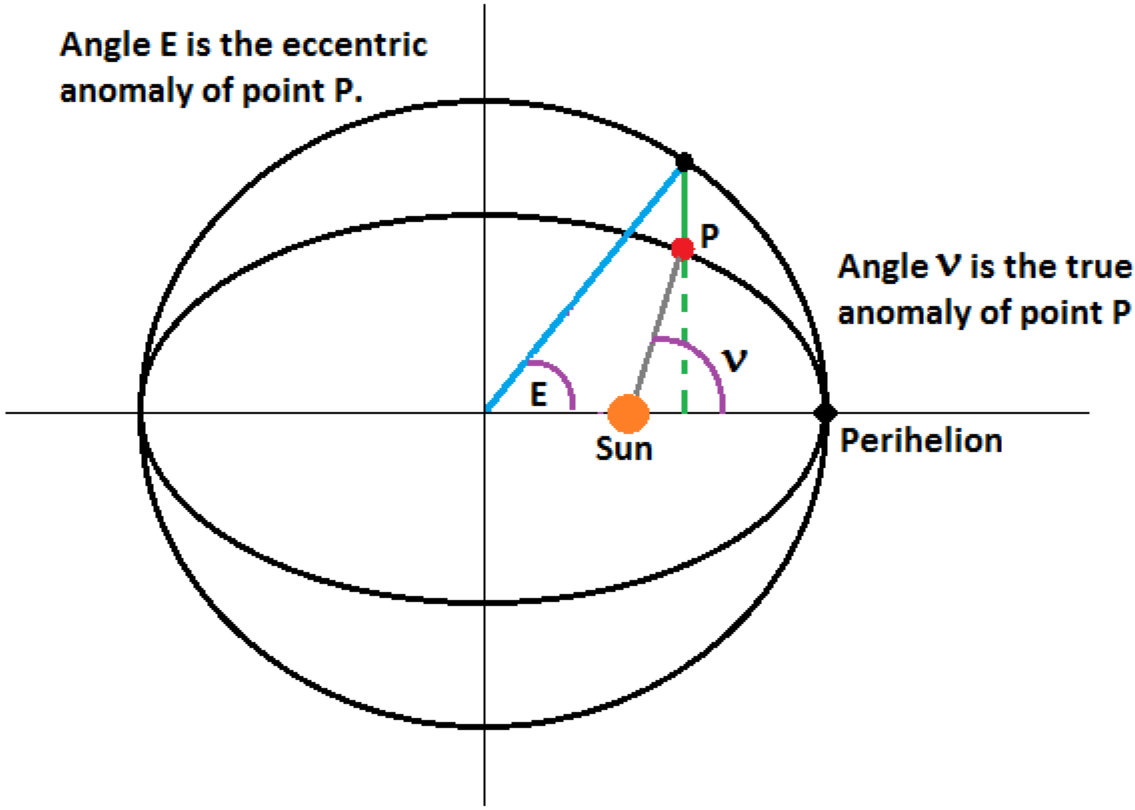


The **Eccentric Anomaly (E)** is related to the **Mean Anomaly (M) by Kepler's Equation :**

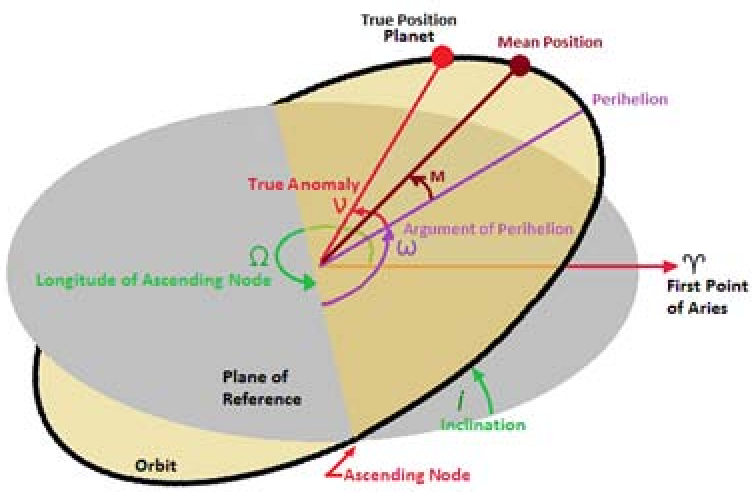
$$M = E - e * \sin(E)$$ where **M** and **E** are in radians

The relation between the **True Anomaly (ν)** and the **Eccentric Anomaly (E)** is:

$$\nu = 2 \arctan\left(\sqrt{\frac{1+e}{1-e}} \tan\frac{E}{2}\right)$$



Now look at it again to see how it all fits. The Sun is at a focal point, not at the center.



If the Sun was at the center then the orbit would be a circle and the **Mean Anomaly,** the **Eccentric Anomaly** and the **True Anomaly** would all be the same angle pointing to the same object on the orbital path. But, this is rarely the case.

As an exercise let's suppose that the orbit of a planet is a perfect circle. The **Mean Anomaly = 360° * t /P**. Let's assume the **Period (P)** = 1000 days and **t** = 125 days. This makes the **Mean Anomaly** = 360 * 125/1000 = 45°

So, the angle (**M**) is given as 45°. Since there is no **Perihelion** nor Aphelion in a circle then if we set **Perihelion** = 0°, this puts the planet at 45° from the **Ascending Node**.

With **M** = 45°, and since **e** = 0 for a circle, and given **M = E - e * sin(E)** ; **E** is also equal to 45°. When we plug in **e** = 0 and **E** = 45° into the equation for the **True Anomaly**, we find that **ν = 45°.** Therefore **M, E,** and **ν** are the same in a circle.

So, as you can see, for the vast majority of cases where the orbit is not a perfect circle, we need to first solve for the **Mean Anomaly (M)**. Second, use **M** to solve for the **Eccentric Anomaly (E)**, and third, use **E** to solve for the **True Anomaly (ν)**.

Kepler's Equation (**M = E - e * sin(E)**) cannot be solved algebraically. It is usually solved by using iteration methods with a computer. This method is called **Newton's method** and involves finding the roots of the equation:

$$f(E) = E - e*\sin(E) - M$$

Note: **E** and **M** are measured in radians. This equation does not work for degrees. To solve for the roots, set **f(E)** = 0

As an example:
Given that **eccentricity** = 0.5 and **Mean Anomaly** = 27° = 0.4712389 radians

After a few iterations (use a computer to try different values for **E**) we find that
E = 0.8453359 radians = 48.43418°
and the **True Anomaly (ν)** = 1.3236527 radians = 78.839°

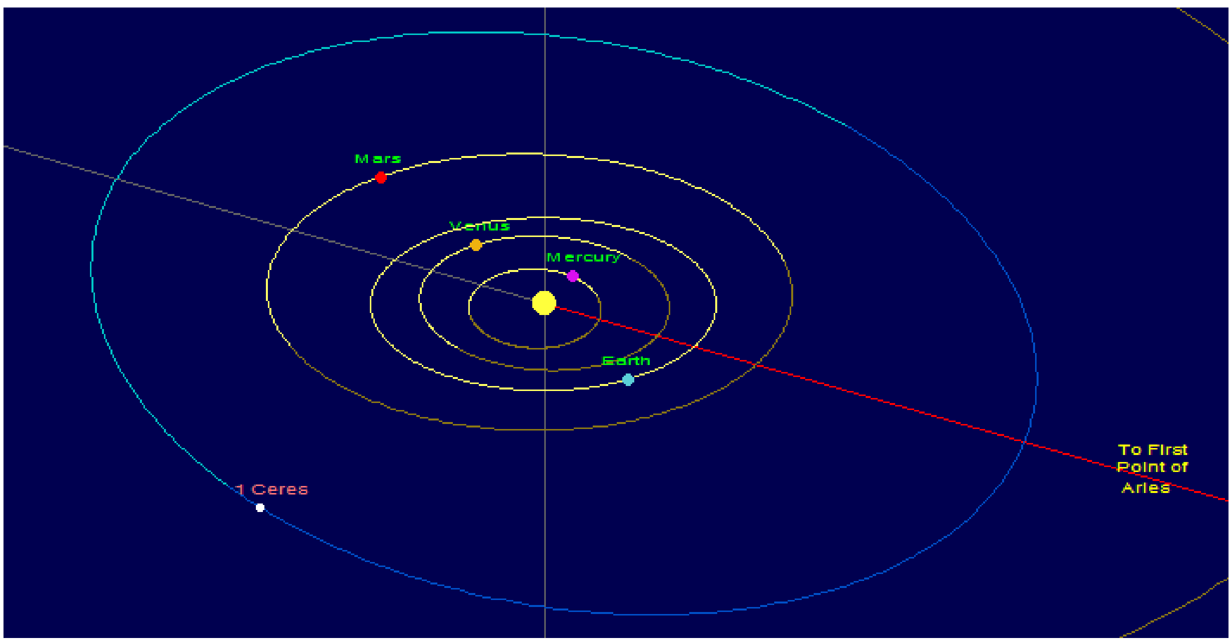
Verify it for yourself: **M** = 0.8453359 - 0.5 * sin(0.8453359) = 0.4712389 radians and
ν = 2 * arctan(sqrt((1+0.5)/(1-0.5)) * tan(0.8453359/2) = 1.3236527 radians

Using Sun, Moon, Planets to Experiment with Orbital Elements

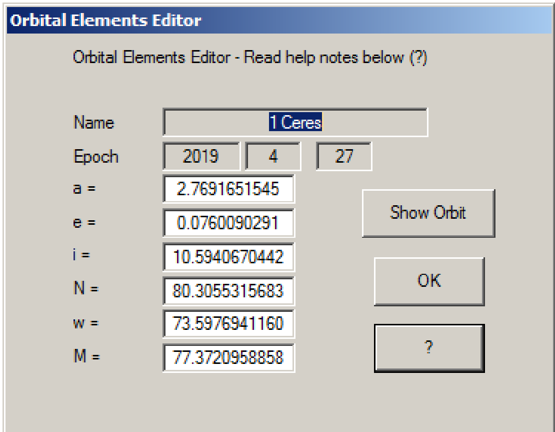
Now that you understand the basic orbital elements let's see how they individually affect the orbit of some object like a planet, asteroid or comet.

If you wish to see exactly the same images as shown below then set the **Date** to 2019/08/22.

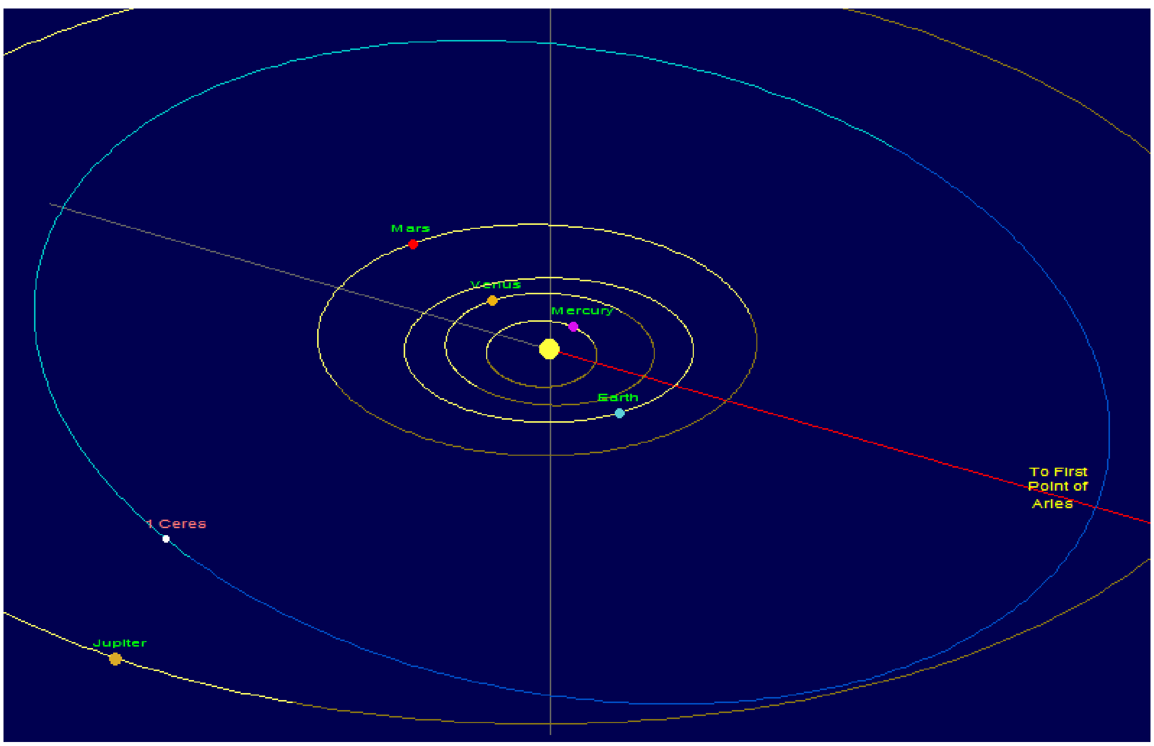
Start by clicking the **View** menu and then **Select Asteroid or Comet for View**. From the **Asteroids** combo box select the first asteroid **Ceres.** You should immediately see the orbit of **Ceres** on the screen.



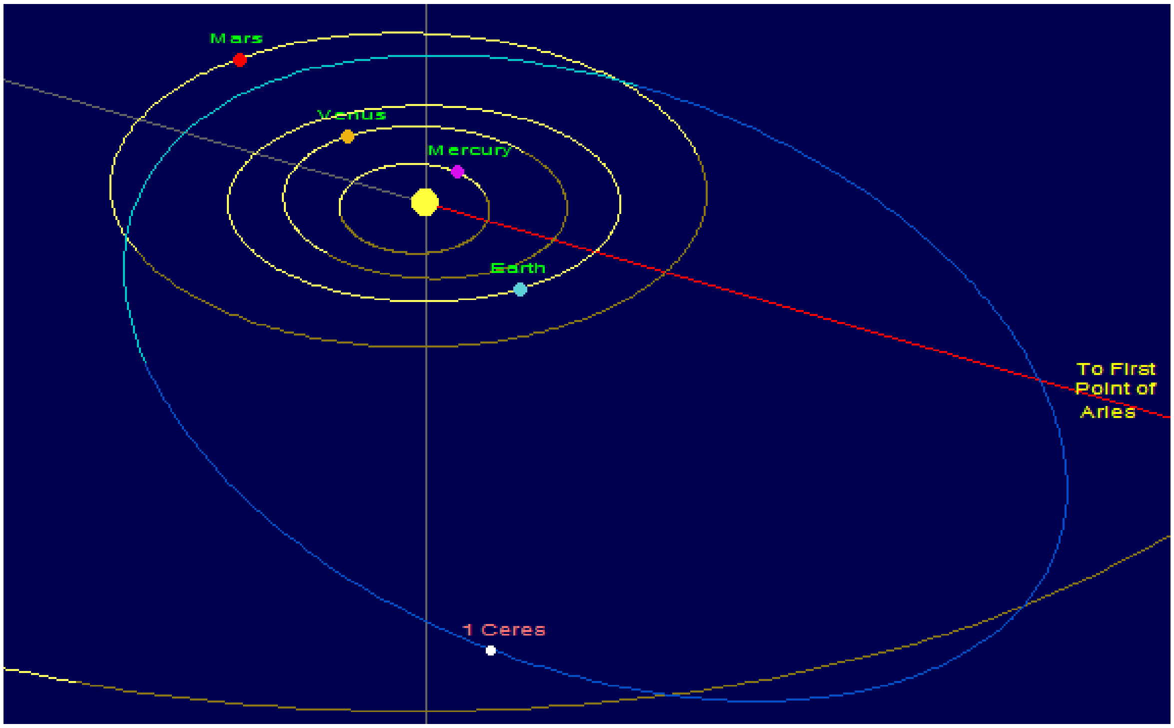
On the dialog box click the button **Edit Orbital Elements** - that brings up another dialog box again showing the orbital elements of **Ceres**, but this time you can make changes to each of the orbital elements and test the changes.



In this dialog box start with the **Semi-Major Axis (a)** and change it from 2.769... to 3.769 (in other words add 1 au), then click the button **Show Orbit**. Check the screen again and notice that the overall orbit size has become noticeably larger. It is closer to the orbit of Jupiter now.



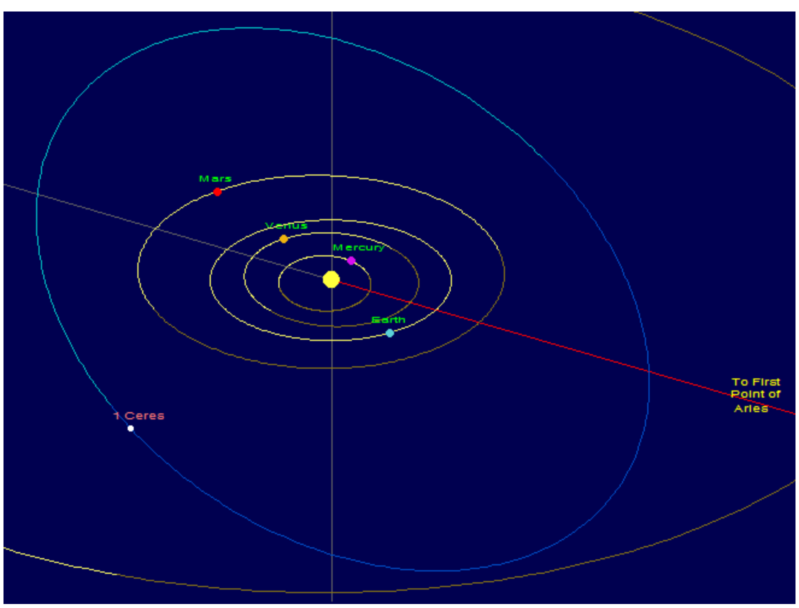
Change the **Semi-Major Axis (a)** back to its original value of 2.769... and then change **Eccentricity (e)** from 0.076009... to 0.576009... (add 0.5). Click **Show Orbit**.



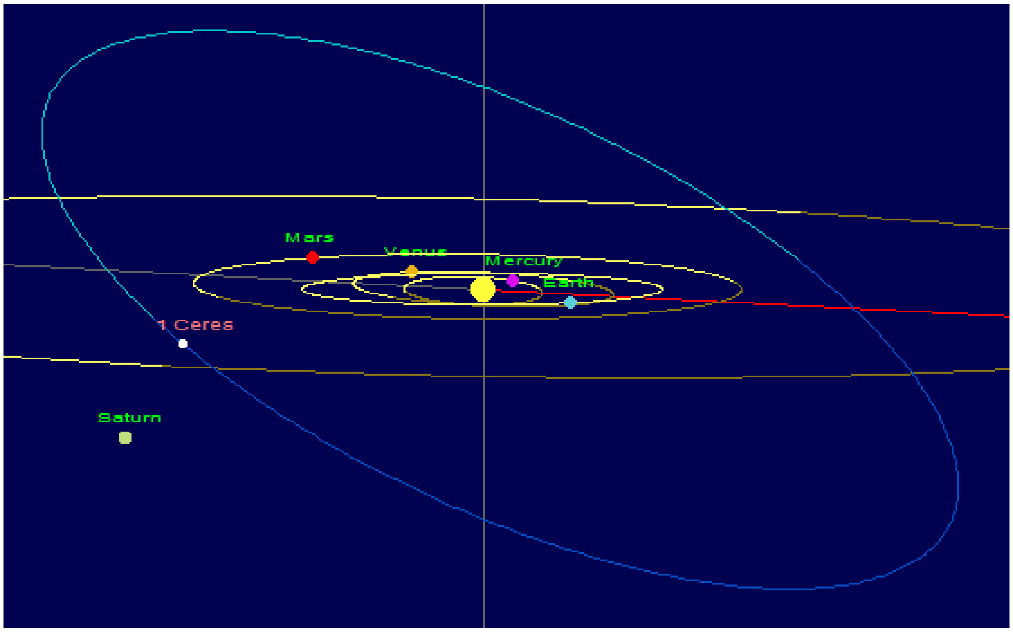
Remember, you may get different results depending on the date that you are doing this.

Change **Eccentricity (e)** back to its original value of 0.076009...

Next change **Inclination (i)** from 10.594... to 30.594... (add 20), **Show Orbit**.

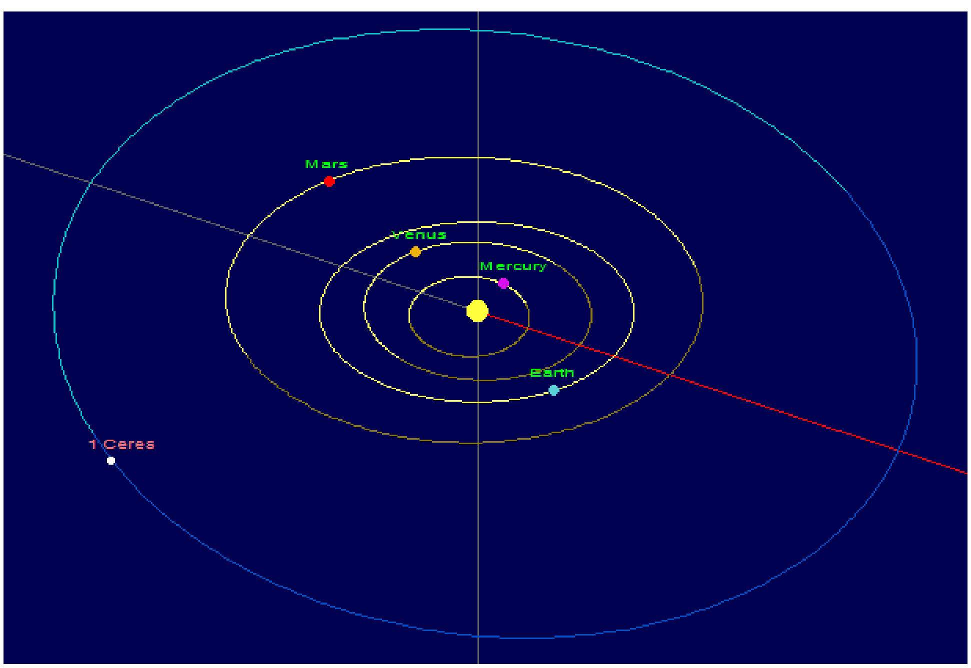


If you didn't notice a significant change in the tilt of the orbit then use the **Vertical Slider** on the right side of the screen to see it better.



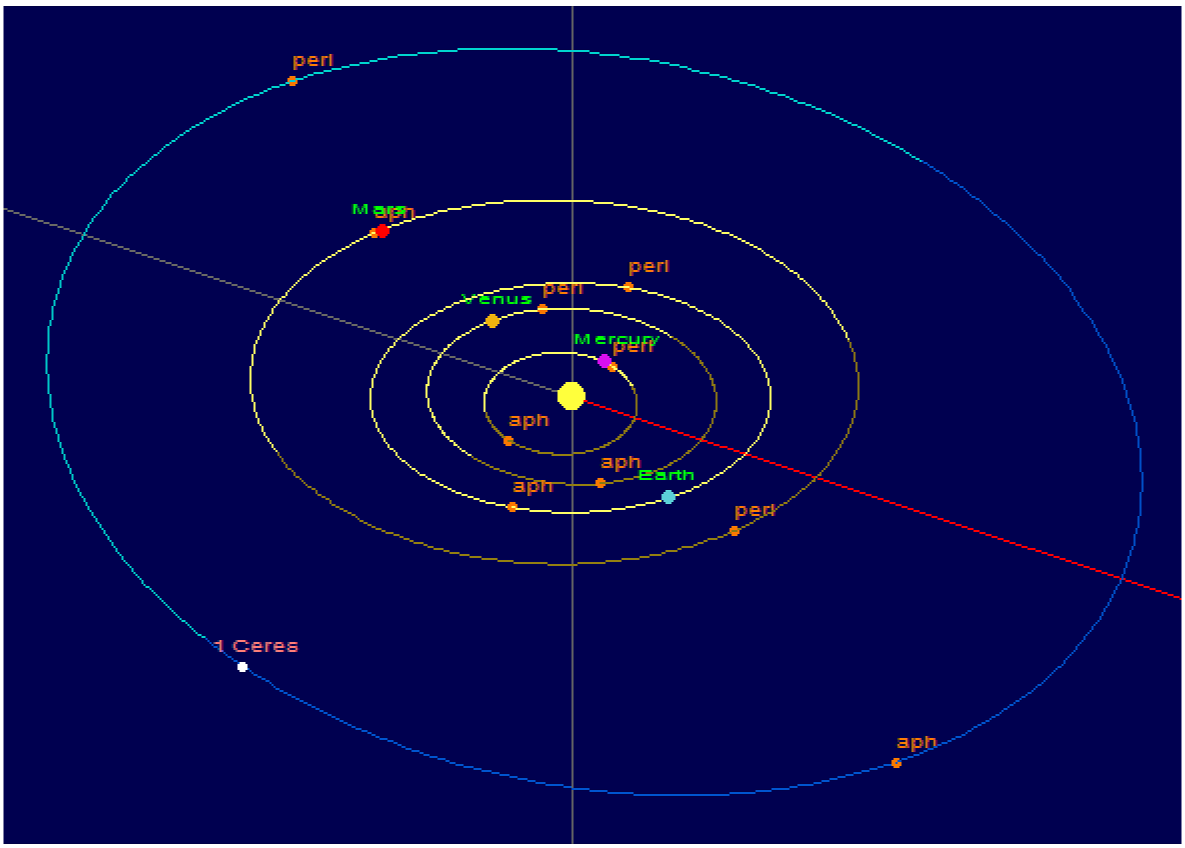
Change the inclination back to where it was (10.594...) and return the slider.

Now let's change the **Longitude of Ascending Node (Ω or N)** from 80.3055... to 60.3055... (subtract 20) and click **Show Orbit**.



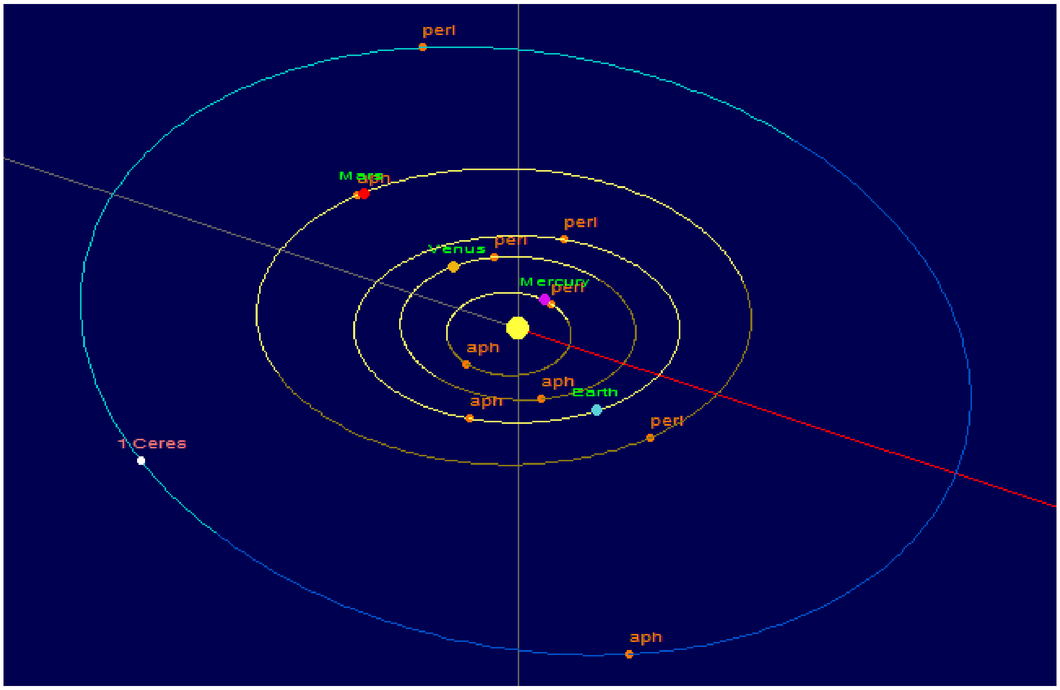
Did you notice that the entire orbit rotated a bit backwards. That's because you have reduced the angle from the **First Point of Aries** to the **Longitude of the Ascending Node.** Change it back to 80.3055...

Next, we will change the **Argument of Perihelion (ω or w)** but first go to the menu item **View** and **Show Perihelion, Aphelion Points** and click it. You should see the points '**peri**' and '**aph**' marked on the orbital path.



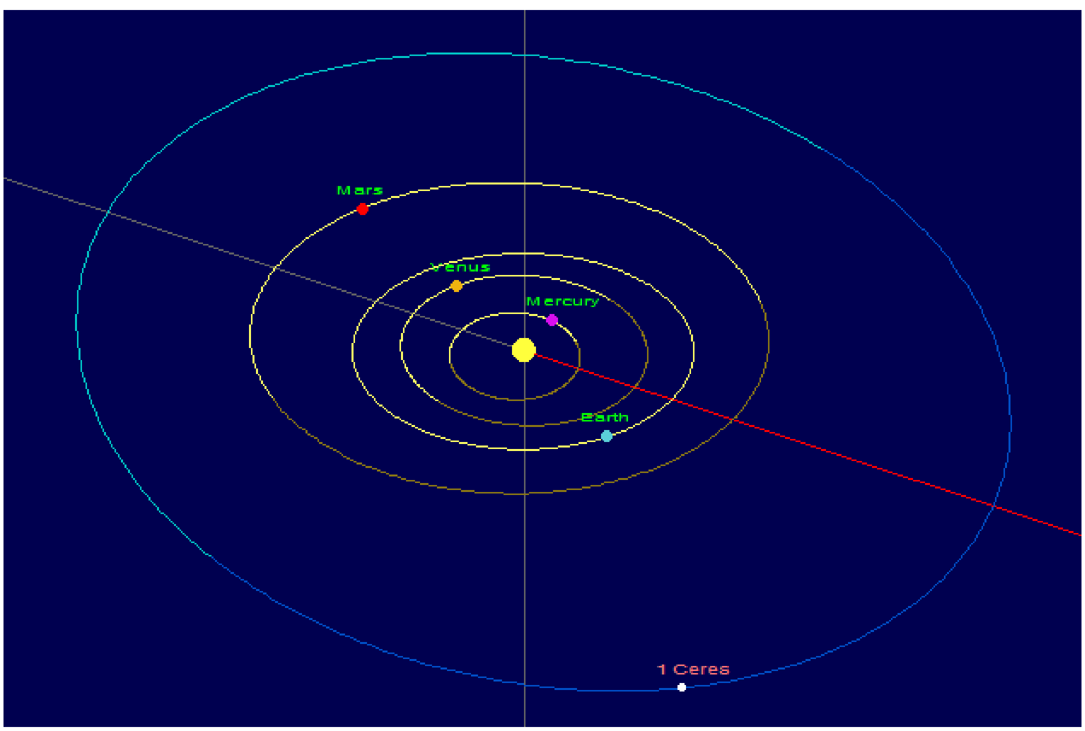
Now continue to make the change to the **Argument of Perihelion** from 73.597... to 53.597... (decrease by 20).

You should notice that **Ceres** dropped back along the orbital path, closer to the point of **Perihelion**.



Change **w** back to 73.597... and turn off the points of '**peri**' and '**aph**'.

Now, for the last orbital element, the **Mean Anomaly (M)**; it is at 77.372...
Change it to 137.372... (add 60).



Notice how the position of **Ceres** moved at a considerable angle forward along the orbital path, as you might expect.

Hopefully, this exercise has cleared up some ideas about orbital elements.

If you have any questions or comments, just send an email to **dgarner@astrotables.com**